SPACE–TIME DEVELOPMENT OF PERTURBATIONS IN THE BOUNDARY LAYER

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According to modern representations, the transition to turbulence in the boundary layer can occur because of flow instability to small perturbations. The initial stage of such a process is described by the linear theory of hydrodynamic stability.

The classical approach to the problem is based on an examination of the elementary wave perturbations [1] in a longitudinally homogeneous flow. That the motion is not parallel is taken into account by corrections to the approximations [2].

Under natural conditions the fluctuations are localized in space. A whole spectrum of oscillations is excited during evolution, and changes occur in space and time.

A problem with initial data simulates such a process. Its investigation invokes great interest and is of practical value for the determination of the transition zone [1-17]. A set of data is obtained about the evolution of narrow wave packets in parallel and slightly nonparallel flows [3-14]. Nevertheless, problems remain that require solution. In particular, the question of the behavior of fluctuations of arbitrary shape in the boundary layer requires clarification. The space–time linear evolution of perturbations in a nonparallel boundary layer is examined in this paper.

Ordinarily the asymptotic behavior of a packet of Tollmien–Schlichting (TS) waves is analyzed in papers without discussing the connection with the initial distribution [1, 3-13]. Meanwhile, such a distribution plays an important part in the instability development law. We illustrate this by the example of a packet of TS-waves in a two-dimensional Blasius-type flow.

In this case the perturbation field is described by a dimensionless stream function ψ which we represent in the form

$$\psi(x, y, t) = \int_{-\infty}^{\infty} d\alpha e^{i\theta(\alpha)}\varphi(y, \alpha) A(\alpha) = \int_{\alpha_0-\delta}^{\alpha_0+\delta} d\alpha e^{i\Delta\theta(\alpha)} A(\alpha)\varphi(y, \alpha) e^{i\theta(\alpha_0)} + J,$$

where $\theta(\alpha) = \alpha x - \Omega(\alpha)t$; $\Omega(\alpha) = \omega(\alpha) + i\gamma(\alpha)$ is the eigenfrequency, $\varphi(y, \alpha)$ is the eigenfunction of the Orr–Sommerfeld problem, α_0 is the central wave number, J is the remainder term, $A(\alpha)$ is the initial amplitude, and

$$\Delta\theta = -t(\alpha_0 - \alpha)\{(\Omega'(\alpha_0) - x/t) + \Omega''(\alpha_0)(\alpha - \alpha_0)/2\},\ d^n\Omega/d\alpha^n = \Omega^{(n)}.$$

The main contribution is introduced by the wave number domain $(\alpha_0 - \delta, \alpha_0 + \delta)$, which is found from the condition

$$\delta^2 t \max(|\omega''(\alpha_0)|, |\gamma''(\alpha_0)|) \lesssim 1.$$

If there is a peak in the distribution at t = 0, say

$$A(\alpha) \simeq A(\alpha_0)\exp\left(-\frac{z^2\Delta\alpha^2}{2}\right), \tag{1}$$

where $\Delta\alpha \sim z^{-1} \ll \alpha_0$, then $\delta \gg \Delta\alpha$ and

$$\psi \simeq e^{i\theta(\alpha_0)}\varphi(y, \alpha_0) A(\alpha_0)\int_{-\delta}^{\delta} d\eta \exp\{-t^2(\Omega'(\alpha_0) - x/t)^2/2z^2 - [z\eta/\sqrt{2} + it(\Omega'(\alpha_0) - x/t)]^2\} = e^{i\theta(\alpha_0)} A(\alpha_0)\varphi(y, \alpha_0)\exp\{-(\Omega'(\alpha_0)t - x)^2/2z^2\}.$$

The case of a spatially localized perturbation is most often examined in the theory of hydrodynamic stability. Then $\Delta\alpha \sim z^{-1} \sim \alpha > \delta$ and the wave packet is separated out because

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of the existence of $\alpha = \alpha_m$, $\gamma(\alpha_m) = \max_\alpha \gamma(\alpha) > 0$. For $t > \gamma^{-1}(\alpha_m) > 1$ $\alpha_0 \simeq \alpha_m$ and

$$\psi \simeq \frac{A(\alpha_m)\,\varphi(y, \alpha_m)}{\sqrt{2\pi t\Omega''(\alpha_m)}} \exp i\left\{\theta(\alpha_m) - \frac{t(\omega'(\alpha_m) - x/t)^2}{2\Omega''(\alpha_m)}\right\} \tag{2}$$

under the conditions

$$|\Omega''(\alpha_m)(\alpha - \alpha_m)| > |\Omega'(\alpha_m) - x/t|, \ |\Omega''(\alpha_m)/t| > |\Omega'(\alpha_m) - x/t|^2. \tag{3}$$

Conditions (3) follow from the requirement of smallness of J and neglect of the deviation of the maximum of the integrand in (2) from $\alpha = \alpha_m$. The method of analysis performed (the Gauss method) is well known in plasma physics [18]. Let us note that its applicability in the theory of hydrodynamic instability was investigated numerically in [5]. The inequalities (3) yield analytic estimates of its accuracy: the solution (2) tends to the exact solution in the circle $L = |x/t - \omega'(\alpha_m)| \to 0$ whose dimensions diminish ($L \sim t^{-1/2}$). Consequently, the Gauss method is applicable for analysis of the evolution of the maximum packet intensity but not its shape.

It follows from (1) and (2) that even in a parallel flow where the wave properties are determined completely by the dispersion equation $\Omega = \Omega(\alpha)$, different initial conditions specify a difference in the evolution laws.

In a nonparallel flow the problem is complicated substantially. There is not complete separation of the transverse and wave structures [6]. The local spectrum is deformed in both α and in x. Formation of a train of localized initial perturbation has no explicit physical basis. For $t \gg 1$ it does not degenerate absolutely into such a wave. Nevertheless, known papers are constrained to the examination of this case and are constructed under the assumption of invariance of the packet carrier frequency [6-8, 12-15]. Investigation of the evolution of boundary layer perturbations of an arbitrary initial kind is the basic aim of this paper.

Let us examine two-dimensional motion. We introduce the dimensionless stream function $\Psi(x, y, t) = \Psi_0(x, y) + \psi(x, y, t)$, where Ψ_0 describes the unperturbed field, and ψ its perturbation

$$\max|\Psi_0|/\max|\psi| \gg 1.$$

To the accuracy of terms nonlinear in ψ, we have

$$\begin{gathered} Z\psi \equiv \left(\frac{\partial}{\partial t}\Delta + \frac{\partial\Psi_0}{\partial y}\frac{\partial}{\partial x}\Delta - \frac{\partial^3\Psi_0}{\partial y^3}\frac{\partial}{\partial x} - \frac{\Delta^2}{\mathrm{Re}}\right)\psi = M(\Psi_0)\,\psi, \\ \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\psi = 0 \ (y = 0), \quad \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\psi < \infty \ (y \to \infty), \\ \psi = \psi(x, y, 0) \ (t = 0), \\ M(\Psi_0) = -\frac{\partial}{\partial x}\Delta\Psi_0\frac{\partial}{\partial y} + \frac{\partial}{\partial y}\frac{\partial^2}{\partial x^2}\Psi_0\frac{\partial}{\partial x}. \end{gathered} \tag{4}$$

Making the quantities dimensionless is performed by the standard method with respect to the free stream velocity U_∞ and the appropriate length l.

Slowness of the longitudinal variation holds:

$$\nu = \max\left|\Psi_0^{-1}\frac{\partial}{\partial x}\Psi_0\right|_{x_0} \sim \mathrm{Re}^{-1}.$$

To the accuracy of $O(\nu)$ we arrive at a linear stability problem in the parallel stream. Its solution can be found by a Fourier–Laplace transform [4, 11]:

$$\begin{gathered} \psi(x, y, t) = \int_{\alpha_0}^{\infty}\left\{\sum_{n=1} A_n(\alpha)\,\varphi(y, \alpha)\,e^{-i\Omega_n t} + \int_0^{\infty} A_{(k)}(\alpha)\,\varphi_{(k)}(y, \alpha)\,e^{-i\Omega_{(k)}t}dk\right\}e^{i\alpha x}d\alpha, \\ A_{n(k)} = -\frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} dy\,dx\,e^{i\alpha x}\Delta\psi(x, y, 0)\,\tilde{\varphi}_{n(k)}(y, \alpha). \end{gathered} \tag{5}$$

Here $\Omega_{n(k)}$, $\varphi_{n(k)}$ are determined by the Orr–Sommerfeld problem, the subscripts n, (k) denote the discrete and continuous spectrum modes, and $\hat{\varphi}_{n(k)}$ is the solution of the adjoint problem.

The condition of applicability of a local parallel approximation $\alpha_0/\nu \gg 1$ is true for a perturbation with $A_{n(k)}(\alpha)$, where $A_{n(k)}(\alpha) \approx 0$ for $|\alpha| < |\alpha_0|$. The wave properties are determined by the dispersion relation

$$\Omega_n(\alpha) = \omega_n(\alpha) + i\gamma_n(\alpha),\ \Omega_{(k)}(\alpha) = -i(\alpha^2 + k^2 + i\alpha \operatorname{Re} U_\infty)/\operatorname{Re} = \omega_{(k)}(\alpha) + i\gamma_{(k)}(\alpha).$$

There is a unique mode (TS), unstable in a certain Re range, in the boundary layer. In principle, damped waves [19], primarily with $|\gamma/\omega| \ll 1$, can even turn out to be important for transition. A change in the main flow velocity profile in the motion direction changes the local perturbation spectrum at scales much greater than $\lambda = \alpha_0^{-1}$, $\partial \ln|\Omega_{n(k)}, \varphi_{n(k)}|/\partial x \sim \nu \ll \alpha_0$.

The solution of (14) can be found by expansion in the parameter $\nu/\alpha_0 \sim \nu \ll 1$:

$$\psi = \sum_n \int_{\alpha_0}^{\infty} d\alpha e^{i\theta_n} a_n(\alpha, \Omega_n(\alpha, \rho), \rho, \tau)\, \varphi_n(y, \alpha, \rho, \Omega_n(\alpha, \rho))$$

$$+ \int_0^\infty \int_{\alpha_0}^\infty dk d\alpha e^{i\theta(k)} a_{(k)}(\alpha, \Omega_{(k)}(\alpha, \rho), \rho, \tau)\, \varphi_{(k)}(y, \alpha, \rho, \Omega_{(k)}(\alpha, \rho)) + \sum_{m=1} \nu^m \psi^{(m)}(y, x, t, \rho, \tau); \tag{6}$$

$$\frac{\partial}{\partial t} a_{n(k)} - \gamma_{n(k)} a_{n(k)} = \sum_{m=1} \nu^m F^{(m)}_{n(k)}; \tag{7}$$

$$\frac{\partial}{\partial t} \theta_{n(k)} = -\omega_{n(k)}(\alpha, \rho) + \sum_{m=1} \nu^m \Phi^{(m)}_{n(k)}; \tag{8}$$

$$\frac{\partial}{\partial x} \theta_{n(k)} = \alpha, \tag{9}$$

where $\rho = x_0 + \nu x$; $\tau = \nu t$; $a_{n(k)} = A_{n(k)} e^{\gamma_{n(k)}(\alpha,\rho)t}$; $\theta_{n(k)}(\alpha, x, t, \rho, \tau)$ is the wave phase, and $\psi^{(m)}$, $F^{(m)}_{n(k)}$, $\Phi^{(m)}_{n(k)}$ are to be determined. Therefore, the solution is constructed in the form of an expansion in waves with fixed wave numbers (α is the independent variable), here slowness of the change in its spectrum parameters is assumed. Such an approach is based on ideas proposed in [20] for one-dimensional conservative systems.

We obtain from (4), (6)-(9) by omitting the subscripts (n, (k)):

$$a\tilde{Z}\left(\frac{\partial}{\partial y}, y, \rho, i\alpha, -i\Omega\right)\varphi(y, \alpha, \rho, \Omega) + \left\{\left(\frac{\partial}{\partial t} a - \gamma a\right)\tilde{Z}_{-i\Omega}\varphi\right.$$

$$\left. + i\Phi^{(1)} a\tilde{Z}_{-i\Omega}\varphi + \nu\frac{\partial}{\partial \rho}(a\tilde{Z}_{i\alpha}\varphi) + \nu M_1(\Psi_0) a\varphi\right\} + \nu e^{\gamma t}\tilde{Z}\hat{\psi}^{(1)}(y, \alpha, \Omega, \rho, \tau) + O(\nu^2) = 0, \tag{10}$$

where

$$\hat{\psi}^{(1)} = \int_0^{2\pi} \psi^{(1)} e^{if} df/2\pi;\ f = \alpha x - \Omega t;\ M_1 = \partial^3\Psi_0/\partial y^2 \partial\rho \cdot \partial/\partial y;$$

$$\tilde{Z}_{-i\Omega} = -\partial\tilde{Z}/\partial i\Omega = \Delta + O(\nu);\ \tilde{Z}_{i\alpha} = \partial\tilde{Z}/\partial i\alpha = \alpha\Omega + \partial\Psi_0\Delta/\partial y$$

$$-2\alpha^2\partial^2\Psi_0/\partial y^2 - \partial^3\Psi_0/\partial y^3 + 4\alpha\Delta/\operatorname{Re};\ \Delta = \frac{\partial^2}{\partial y} - \alpha^2.$$

The approximations of (ρ, τ) are main fixed; $(\alpha, \Omega, \varphi)$ are determined by solving the Orr–Sommerfeld problem $Z\varphi = 0$ (5). To $O(\nu)$ accuracy $\psi^{(1)}$ has bounded values under the condition of orthogonality of $\tilde{\varphi}$ in the expression in the braces in (10):

$$\langle\tilde{Z}_{-i\Omega}\varphi\rangle(F^{(1)} + i\Phi^{(1)}) + \left(\frac{\partial}{\partial\rho} a + \frac{\partial\Omega}{\partial\rho}\frac{\partial}{\partial\Omega} a\right)\langle\tilde{Z}_{i\alpha}\varphi\rangle +$$

$$+ \left\langle\left(\frac{\partial}{\partial\rho}\tilde{Z}_{i\alpha}\right)\varphi + M_1\varphi\right\rangle a + \left\langle\tilde{Z}_{i\alpha}\left(\frac{\partial}{\partial\rho} + \frac{\partial\Omega}{\partial\rho}\frac{\partial}{\partial\Omega}\right)\varphi\right\rangle a = 0,$$

$$\langle f_r\rangle = \int_0^\infty \tilde{\varphi}_r(y, \alpha) f_r dy.$$

Separating real and imaginary parts, we determine $F^{(1)}$, $\Phi^{(1)}$. Then to $O(\nu^2)$ accuracy, we have for each (n, (k); α)

$$\frac{\partial a}{\partial t} - \gamma a + \frac{\partial\omega}{\partial\alpha}\frac{\partial a}{\partial x} + \frac{\partial\omega}{\partial x}\frac{\partial\Omega}{\partial\alpha}\frac{\partial a}{\partial\Omega} + H_r = 0; \tag{11}$$

$$\Phi^{(1)} = -\frac{1}{a}\left\{\frac{\partial\gamma}{\partial\alpha}\frac{\partial}{\partial x} + \frac{\partial\gamma}{\partial x}\frac{\partial}{\partial\alpha} + H_i\right\} a;\ a(x, \alpha) = a|_{t=0}, \tag{12}$$

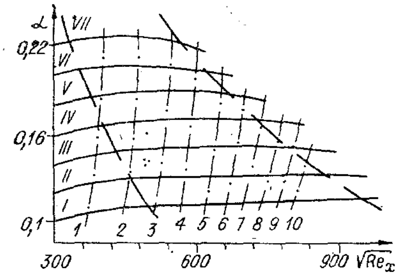

Fig. 1

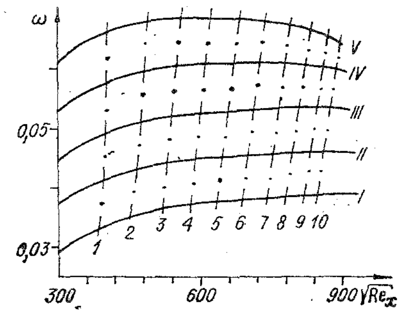

Fig. 2

where

$$\frac{\partial}{\partial\alpha}\Omega = -\langle\tilde{Z}_{i\alpha}\varphi\rangle/\langle\tilde{Z}_{-i\Omega}\varphi\rangle;$$

$$H = H_r + iH_i = \Big\langle \alpha\varphi\frac{\partial\Omega}{\partial x} - \frac{\partial^2\Psi_0}{\partial y\partial x}\Delta\varphi + 2\alpha^2\varphi\frac{\partial^2\Psi_0}{\partial y\partial x} - \frac{\partial^4\Psi_0}{\partial x\partial y^3}\varphi + \frac{\partial^3\Psi_0}{\partial x\partial y^2}\frac{\partial\varphi}{\partial y}$$
$$+\left(\alpha\Omega + \frac{\partial\Psi_0}{\partial y}\Delta - 2\alpha^2\frac{\partial\Psi_0}{\partial y} - \frac{\partial^3\Psi_0}{\partial y^3} + 4\alpha\Delta\frac{1}{\mathrm{Re}}\right)\frac{d\varphi}{dx}\Big\rangle\langle\tilde{Z}_{-i\Omega}\varphi\rangle^{-1}.$$

System (11), (12) describes the space–time evolution of perturbations in an inhomogeneous stream. The equations are valid for an arbitrary kind of initial distribution and, in contrast to the case of quasiharmonic packets [7-9, 12-15, 19], contain derivatives with respect to α ($\partial\Omega/\partial x\cdot\partial\Omega/\partial\alpha\cdot\partial a/\partial\Omega = \partial\Omega/\partial x\cdot\partial a/\partial\alpha$). The spectrum is transformed in the space (x, t, α) of real variables (compare with [6, 7]). From the known $F^{(1)}$, $\Phi^{(1)}$ the $\psi^{(1)}$ is determined. Repeating the procedure for $F^{(2)}$, $\Phi^{(2)}$, we find $\psi^{(2)}$, etc. Here $\psi^{(m)}$ is the solution of the inhomogeneous Orr–Sommerfeld equation, while the operators $F^{(m)}$, $\Phi^{(m)}$ contain the terms $\partial^m a/\partial x^{m-j}\partial\alpha^j$.

The condition for the existence of trains in a nonparallel stream can be obtained from (11). If the initial perturbation is represented by a narrow packet (in α) $a \sim \exp(-\Delta\alpha^2 z^2/2)$, $\Delta\alpha \sim z^{-1} \sim \mu \ll 1$, then

$$\frac{\partial a}{\partial\alpha} \sim \frac{\partial a}{\partial\omega} \sim za \sim \mu^{-1} \text{ and } 1 > \frac{\partial a}{\partial\omega}\frac{\partial\omega}{\partial x}\frac{\partial\omega}{\partial\alpha} \sim \frac{\nu}{\mu} > \nu. \tag{13}$$

Then in a first approximation (11) takes the form $\partial a/\partial\alpha = 0$, $a = a(\omega)$, and the perturbation is represented by a quasiharmonic wave with $\omega_S = \omega(\alpha, \rho) = \mathrm{const}$, $\psi = a_S\varphi_S(y, \rho)\exp i\theta_S$ and the phase $\theta_S = -\omega_S t + \int\alpha_S dx$, $\alpha_S = \alpha(\omega_S, \rho)$.

The problem reduces to taking account of the influence of the nonparallel behavior ($\nu > 0$) and the finiteness of the spectrum width of the train ($\mu^{-1} < \infty$) on its evolution in the space (x, t) [7, 8, 12, 13, 21]. The solution can be found by an expansion in the two parameters (μ, ν) [8, 12], it has meaning under the condition (13) $\nu < \mu$. This latter means that the packet width $z \sim \Delta\alpha^{-1}$ should remain greater than the characteristic $\lambda = \alpha^{-1}$, but much less than the scale of the inhomogeneity ν^{-1} [18].

The characteristics $dx/dt = \partial\omega/\partial\alpha$, $d\alpha/dt = \partial\omega/\partial x$; $da/adt = \gamma - H_r$ of (11) differ from the case of the trains [7, 13, 21] because of the absence of a conservation law for ω.

Results of computations in a certain range of motion parameters are given in Figs. 1-4.

The form of the trajectories $\alpha_j = \alpha(\alpha_{j_0}, x)$ for $d\alpha/dx = (\partial\omega/\partial x)(\partial\omega/\partial\alpha)^{-1}$, $\sqrt{\mathrm{Re}_{x_0}} = 300$, $\alpha_j(x_0, \alpha_{j_0}) = \alpha_{j_0} = 0.1$; 0.12; 0.14; 0.16; 0.18; 0.20; 0.22 is shown in Fig. 1 (lines I-VII, respectively). The dashed lines superpose the location of the neutral curves of the quasi-nonparallel theory (making the results dimensionless is performed in the scale $U_\infty\delta^2/\nu$ with $\delta^2 = \nu x/U_\infty$ so that $\mathrm{Re} = \delta U_\infty/\nu = \sqrt{\mathrm{Re}_x} = 600$). The lines $n = 0, 1, 2, \ldots, 10$ note the location of points at different times $t_n = n0.125$ under the initial conditions $\alpha = \alpha_{j_0}$, $\sqrt{\mathrm{Re}_x} = \sqrt{\mathrm{Re}_{x_0}}$ ($n = 0$).

The corresponding behavior of ω and $S = \ln a$ along $\alpha_j(\alpha_{j_0}, x)$ is represented in Figs. 2 and 3; the location of neutral growth points is no different, in practice, from those given by the "nonparallel" theory [2]. The spectrum deformation (in α) at different times t_n (the lines n–n) is characterized by the dependence $S(t) = S(\alpha(t))$ represented in Fig. 4. A shift to the long-wave oscillation domain is noted.

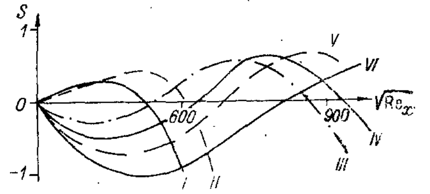

Fig. 3

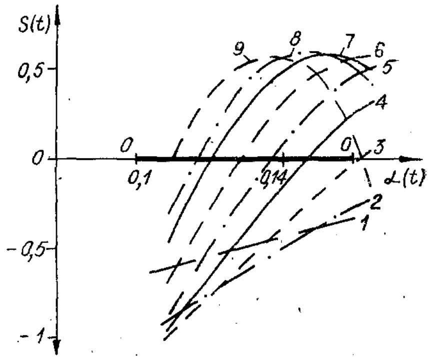

Fig. 4

The case of spatial (three-dimensional perturbations) of the stream is easily generalized within the scope of this approach.

Therefore, the field of an arbitrarily perturbed boundary layer can be traced downstream. On the other hand, the initial spectrum is restored by a distribution known in $x > x_0$. This former can be used to estimate the transition domain. In fact, if the characteristic motion parameters are known in the transition zone, then the spectrum restored upstream can be compared with the real (measured) spectrum and the critical trajectory $\alpha_j(x, \alpha_{j_0})$ and the value x_t of the transition are indicated. It is here understood that nonlinear effects are substantial in small intervals x while turbulization is due to the "internal" development of instability [2, 16, 17]. The critical intensity of each value of $\omega(\alpha_{j_0}, x_0)$ can be tabulated for each class of flows.

Possible considerations about the conditions in the transition zone can be formulated on the basis of representations about two kinds of processes [2, 22-24]. Data are obtained in [23] about a transition associated with the excitation of spatial harmonics of TS waves. Another type (Klebanovskii) is, in our opinion, a result of wave development with a nonlinear critical layer and their subsequent destruction [24, 25]. It is important that the nonlinear phenomena appear effectively in the neighborhood of the upper branch of the neutral stability curve $\alpha_k = \alpha(x)$ in both cases for the oscillation intensity $O(10^{-2})$. Selecting $a(x_k, \alpha_j(x_k)) \sim 10^{-2}$, we find $a(x_0, \alpha_{j_0}) = a_j(x_k) \exp - S_j(x_0, x_k)$ and $\omega(x_0, \alpha_{j_0})$ from (20), (21) in the section x_0. Comparing the measured and computed critical spectrum, we determine the trajectory $\alpha_j(x)$ and its point of intersection $\alpha_k(x_k) = \alpha_j(x_k, \alpha_{j_0})$. In a known sense such an approach combines the "method of e^n" [2, 16], where the growths (but not the amplitudes) are computed to a certain location on the upper branch of the neutral curve, and the "amplitude method" relating the transition point just to the magnitude of the intensity [2, 17]. Evidently, it is desirable to establish a correspondence between the initial spectrum and its sources, i.e., the solution of the susceptibility problem (see [2, 15], say), for "closedness" of application of the criterion.

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